The theory of generalized functions has a long and successful history and is today an indispensable tool
in many branches of mathematics, most notably in the theory of partial differential equations (PDE), harmonic analysis, and mathematical physics. Its linear branch, the theory of distributions, initiated by
S. Sobolev and L. Schwartz, supplies both the language and a repository of techniques and methods for analyzing and solving linear PDEs, most notably the theory of pseudodifferential and Fourier integral operators, and the resulting microlocal analysis.

Starting from the early 1980s, a nonlinear theory of generalized functions has been developed by J.F. Colombeau and his coworkers. This theory provides a framework for addressing nonlinear problems in the presence of singularities, by embedding the space of Schwartz distributions into suitable algebras of generalized functions (Colombeau algebras) that possess optimal permanence property with respect to classical operations. In particular, differentiation and the product of smooth functions are preserved under the embedding. The basic idea of Colombeau's construction is the regularization of distributions through convolution with a mollifier and the expression of analytic properties by asymptotic estimates in terms of a regularization parameter.

Colombeau algebras quickly found manifold applications to problems in partial differential equations involving singularities and/or nonlinearities. In addition, the theory has been successfully applied to several other fields, including low-regularity differential geometry, general relativity, or nonstandard analysis.

The aim of this project is to advance regularity theory in algebras of generalized functions by introducing new mathematical tools into the field, while at the same time integrating various branches of the existing theory into one common line of research. The project will address regularity issues in the nonlinear theory of generalized functions from three interconnected vantage points: algebraic, spectral theoretical, and using Fréchet techniques. The ultimate aim of this line of research is to obtain a satisfactory theory of the geometrical propagation of singularities, as modelled by Fourier integral operator methods, for nonsmooth hyperbolic problems.

The core team of the proposed project will consist of Shantanu Dave, Michael Kunzinger, Eduard Nigsch, and Hans Vernaeve, all of whom are experienced researchers who over last years have made substantial contributions to the field.